Magnetization 'M' & Magnetic field 'H':—

Material is Composed of Atoms. each atom consists of electrons, orbiting around a positive nucleus. the electrons also rotate about its on axis.

Both these motion of electron produces internal magnetic field that are similar to magnetic field produce by a current loop. The current loop has a magnetic moment 'm' $= S\, I_b\, \hat{a}_n$

S - Area of loop, I_b - bound current.

In the absence of external magnetic field the net magnetic moment is zero due to Random orientation. When a magnetic field is applied the net electrons aligned themselves with the field so ~~that~~ net magnetic moment is different from zero.

The magnetization of 'M' is the magnetic dipole moment per unit volume. If there are 'N' atoms in a given volume ΔV. and Kth atom has a magnetic moment m_K then magnetization is given by this expression:

$$\boxed{M = \lim_{\Delta V \to 0} \frac{\sum_{K=1}^{N} m_K}{\Delta V}} \qquad \text{——— (1)}$$

~~if M is~~ A medium ~~for~~ which M is not zero. is said to be magnetized for a volume dv' the magnetic moment is

$$dm = M\, dv' \qquad \text{——— (A)}$$

$$\boxed{m = \int M\, dv'} \qquad \text{——— (B)}$$

~~The~~ The magnetic vector potential due to dm

$$d\vec{A} = \frac{\mu_0 \vec{M} \times \hat{a}_R}{4\pi R^2}\, dv'$$

$$\boxed{d\vec{A} = \frac{\mu_0 \vec{M} \times \vec{R}}{4\pi R^3}\, dv'} \qquad \text{——— (2)}$$

We may write, $\nabla\left(\frac{1}{R}\right) = \frac{-\vec{R}}{R^3}$

but replace $-\nabla\left(\frac{1}{R}\right) = \nabla'\left(\frac{1}{R}\right) = \frac{\vec{R}}{R^3}$

there fore eq^n (2) is convert it

$$\vec{A} = +\frac{\mu_0}{4\pi}\int \vec{M} \times \nabla'\left(\frac{1}{R}\right) dv' \quad \text{--- (3)}$$

<u>using, vector identity</u>

$$\vec{M} \times \nabla\left(\frac{1}{R}\right) = \frac{1}{R}(\vec{\nabla} \times \vec{M}) - \vec{\nabla} \times \frac{\vec{M}}{R}$$

but this in eq^n (3)

$$\vec{A} = \frac{\mu_0}{4\pi}\int\left[\frac{1}{R}(\vec{\nabla} \times \vec{M}) - \vec{\nabla} \times \frac{\vec{M}}{R}\right] dv'$$

$$\vec{A} = \frac{\mu_0}{4\pi}\int_{v'} \frac{\vec{\nabla} \times \vec{M}}{R} dv' - \frac{\mu_0}{4\pi}\int_{v'} \vec{\nabla} \times \frac{\vec{M}}{R} dv' \quad \text{--- (i)}$$

again using another vector identity

Convert volume integral into surface integral

$$\int_{v'} (\vec{\nabla} \times \vec{F}) dv' = \int_s \vec{F} \times d\vec{s}$$

then by (i)

$$\vec{A} = \frac{\mu_0}{4\pi}\int_{v'} \frac{\vec{\nabla} \times \vec{M}}{R} dv' + \frac{\mu_0}{4\pi}\int_s \frac{\vec{M} \times \hat{a}_n}{R} ds' \quad \text{--- (4)}$$

$$\vec{A} = \frac{\mu_0}{4\pi}\int \frac{\vec{J_b}}{R} dv' + \frac{\mu_0}{4\pi}\int_{s'} \frac{\vec{K_b}}{R} ds' \quad \text{--- (5)}$$

where

$$\boxed{\vec{J_b} = \vec{\nabla} \times \vec{M}} \quad \& \quad \boxed{\vec{K_b} = \vec{M} \times \hat{a}_n} \quad \text{--- (6)}$$

eqn (5) shows that, the potential of a magnetic body is due to a Volume Current density $\vec{J_b}$ and a surface current density $\vec{K_b}$ on the surface of a body

$\vec{J_b}$ is bound volume current density or magnetization volume current density in Amp/m^2.

$\vec{K_b}$ is bound surface current density in Ampere/meter. & $\hat{a_n}$ is unit vector normal to surface.

We have,

$$\vec{J} = \vec{\nabla} \times \vec{H} \quad \& \quad \vec{J_b} = \vec{\nabla} \times \vec{M}$$

$\vec{M}$ is analogus to $\vec{H}$ and both have the same units. In free space

$M = 0$. that means

$$\vec{\nabla} \times \vec{H} = \vec{J_f}$$

or,

$$\boxed{\vec{\nabla} \times \frac{\vec{B}}{\mu_0} = \vec{J_f}}$$

$\vec{J_f}$ is free current volume density.

In a material medium,

$M \neq 0$

Total current density $= \vec{J_f} + \vec{J_b}$

and, $\vec{\nabla} \times \frac{\vec{B}}{\mu_0} = \vec{J_f} + \vec{J_b} = \vec{J}$

$$\vec{\nabla} \times \frac{\vec{B}}{\mu_0} = \vec{\nabla} \times \vec{H} + \vec{\nabla} \times \vec{M}$$

using vector Identity

or, $\boxed{\vec{B} = \mu_0 (\vec{H} + \vec{M})}$ ——— (7)

For linear materials $\vec{M}$ depends linearly on $\vec{H}$. (which does not show B-H curve) Such that,

$$\boxed{\vec{M} = \chi_m \vec{H}}$$

'χ_m' is called magnetic suscepetibility of the it is a measure of How sensitive is the material to a magnetic field.

put value of $\vec{M}$ in eqn (7)

then by (7)

$$\vec{B} = \mu_0 (\vec{H} + \chi_m \vec{H})$$

$$\boxed{\vec{B} = \mu_0 \vec{H} (1 + \chi_m)}$$

and,

$$\vec{B} = \mu_0 \cdot \mu_r \vec{H}$$

where, $\mu_r = 1 + \chi_m$

$$\boxed{\mu_r = \frac{\mu}{\mu_0}}$$

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μ is Called permeability of material and μ_r is Called relative permeability of the material.

The ratio of permeability of a given material [illegible] to that of free space.